Principles of Finance

Prof. Dr. Dennis A. V. Dittrich

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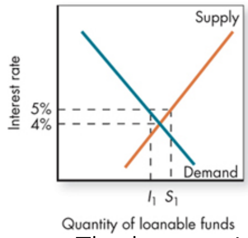
The Role of Interest Rates in the Financial Sector

- The **interest rate** is the price paid for use of a 8nancial asset
- The long-term interest rate is the price paid for 8nancial assets with long maturities,
  - The market for long-term pnancial assets is called the **loanable funds market**



At equilibrium, the quantity of loanable funds supplied (savings) is equal to the quantity of loanable funds demanded (investment)

- The short-term interest rate is the price paid for 8nancial assets with short maturities,
  - Short-term pnancial assets are called **money**

Interest Rates

A rate of interest (growth, in=ation, etc.) is not properly de8ned unless we state the time period to which it applies and the method of compounding to be used.
2% per annum is very die erent from 2% per month.

The e–ect of an interest rate of 10% per annum at di–erent frequencies of compounding.

$$
\begin{array}{lll}
\text{Annually} & £100 & \rightarrow \quad £110 \\
\text{Semi-annually} & £100 & \rightarrow \quad £100 \times (1.05)^2 = £110.25 \\
\text{Quarterly} & £100 & \rightarrow \quad £100 \times (1.025)^4 = £110.381\ldots \\
\text{Monthly} & £100 & \rightarrow \quad £100 \times \left(1+\frac{.10}{12}\right)^{12} = £110.471\ldots \\
\text{Weekly} & £100 & \rightarrow \quad £100 \times \left(1+\frac{.10}{52}\right)^{52} = £110.506\ldots \\
\text{Daily} & £100 & \rightarrow \quad £100 \times \left(1+\frac{.10}{365}\right)^{365} = £110.515\ldots \\
\text{Continuously} & £100 & \rightarrow \quad £100 \times e^{0.10} = £110.517\ldots
\end{array}
$$

Discrete compounding

$$P_t = \left(1 + \frac{r}{n}\right)^{nt} P_0$$

We can solve this equation for any of the 8ve quantities given the other four

a) present value
b) 8nal value
c) implicit rate of return
d) time
e) interval of compounding

Continuous Compounding

$$\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^n = e^r$$

$$P_t = e^{rt} P_0$$

We can solve this equation for any of the four quantities given the other three

a) present value
b) 8nal value
c) implicit rate of return
d) time

Net Present Value

$$NPV = \sum_{0}^{T} \frac{P_t}{(1+r)^t}$$

P_t Cash =ow in period t

r discount rate

The present value of an expected future payment ....?.... as the interest rate increases.

With an interest rate of 6%, the present value of $100 next year is approximately...?

Yield to Maturity , Internal Rate of Return

The interest rate that equates the present value of cash =ow payments received from a debt instrument with its value today

If $P_0, P_1, ..., P_T$ is the stream of cash =ows, the internal rate of return is the solution of the polynomial of degree T obtained by setting the NPV equal to zero

$$\sum_{0}^{T} \frac{P_t}{(1+r)^t} = 0$$

The polynomial de8ning the IRR has T (complex) roots. Generally, there is only one meaningful real root to this polynomial equation, in other words one corresponding to an IRR.

Four Types of Credit Market Instruments

- Simple Loan
  - A credit market instrument that provides the borrower with an amount of funds that must be repaid at the maturity date along with an interest payment
- Fixed Payment Loan
  - A credit market instrument that requires the borrower to make the same payment every period until the maturity date
- Coupon Bond
- Discount Bond

Simple Loan

$$\text{PV} = \text{amount borrowed} = \$100$$
$$\text{CF} = \text{cash flow in one year} = \$110$$
$$n = \text{number of years} = 1$$
$$\$100 = \frac{\$110}{(1+i)^1}$$
$$(1+i)\,\$100 = \$110$$
$$(1+i) = \frac{\$110}{\$100}$$
$$i = 0.10 = 10\%$$

For simple loans, the simple interest rate equals the yield to maturity

Fixed Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}$$

Bonds

- A bond is a sophisticated IOU that documents who owns how much and when payment must be paid.
- Issuing bonds allows borrowing directly from the public.
- Lender: one who buys a bond
- Borrower: one who issues a bond
- Corporations and governments at all levels borrow money by issuing bonds.
- All bonds involve a risk.
  - Major issues are graded by rating companies: Standard and Poor's, Moody's
  - Grades range from lowest risk (AAA) bonds in current default (D)
  - The higher the risk the greater the interest rate required to get lenders to buy the bonds.

Coupon Bond

A **coupon** payment on a bond is a periodic interest payment that the bondholder receives during the time between when the bond is issued and when it matures.

Coupon Bond

Using the same strategy used for the fixed-payment loan:

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity date

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate
- The price of a coupon bond and the yield to maturity are negatively related:

  The yield to maturity is greater than the coupon rate when the bond price is below its face value

Which of the following $5,000 face-value securities has the highest yield to maturity?

a) A 6 percent coupon bond selling for $5,000

b) A 6 percent coupon bond selling for $5,500

c) A 10 percent coupon bond selling for $5,000

d) A 12 percent coupon bond selling for $4,500

Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays 8xed coupon payments forever

$$P = \sum_{t=1}^{\infty} \frac{C}{(1+i)^t} = C/i$$

P Price of consol
C yearly interest payment
i yield to maturity

$$i = C/P$$

- For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity

The present value of a consol equals the coupon payment

a) times the interest rate.
b) plus the interest rate.
c) minus the interest rate.
d) divided by the interest rate.

A consol paying $20 annually when the interest rate is 5% has a price of ...?

Discount Bond c Zero Coupon Bond

- No interest payments (=coupons)
- Sold for a price below face value
- US Treasury Bills are examples of such Zero Coupon Bonds
- Some zero coupon bonds are in=ation indexed, the nominal face value is in=ation adjusted
- Short term zero coupon bonds are called Bills

Discount Bond

For any one year discount bond

$$i = \frac{F - P}{P}$$

F = Face value of the discount bond

P = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.

If a $10,000 face-value discount bond maturing in one year is selling for $5,000, then its yield to maturity is...?

Interest Rates vs Rate of Returns

Rate of Return

The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

RET = return from holding the bond from time t to time $t+1$

P_t = price of bond at time t

P_{t+1} = price of the bond at time $t+1$

C = coupon payment

$$\frac{C}{P_t} = \text{current yield} = i_c$$

$$\frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g$$

What is the return on a 5% coupon bond that initially sells for $1,000 and sells for $1,200 next year?

The Distinction Between Interest Rates & Returns

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change
- The more distant a bond's maturity, the lower the rate of return that occurs as a result of an increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise

One-Year Returns on Di–erent-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator using Equation 3.

Interest-Rate Risk

The risk that an investment's value will change due to uncertain future interest rates (a change in the absolute level of interest rates, in the spread between two rates, or in any other interest rate relationship that in=uences the value of an (8nancial) asset).

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period

The Distinction Between Real & Nominal Interest Rates

- Nominal interest rate makes no allowance for in=ation
- Real interest rate is adjusted for changes in price level so it more accurately re=ects the cost of borrowing
- Ex ante real interest rate is adjusted for expected changes in the price level
- Ex post real interest rate is adjusted for actual changes in the price level

Fisher Equation

$$i = i_r + \pi^e$$

i = nominal interest rate

i_r = real interest rate

π^e = expected inflation rate

When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.

The real interest rate is a better indicator of the incentives to borrow and lend.

Fisher Equation

- 2 individuals write a loan contract to borrow P_t dollars at a nominal interest rate of i
- next year the amount to be repaid will be $P_t \times (1 + i)$
- imagine the individuals decide to write a loan contract to guarantee a constant real return r (payment in goods next year instead of cash)
- to repay the loan, the lender has to buy (1+r) units of goods next year for each unit of goods that he can buy now
- the (nominal) prices will change with the inflation π.
- if the price of one unit of goods is P_t today, its price P_{t+1} next year will be $P_{t+1} = P_t \times (1 + \pi)$
- the total amount of dollars needed next year to repay the loan is then $P_t \times (1 + \pi) \times (1 + r)$

Fisher Equation

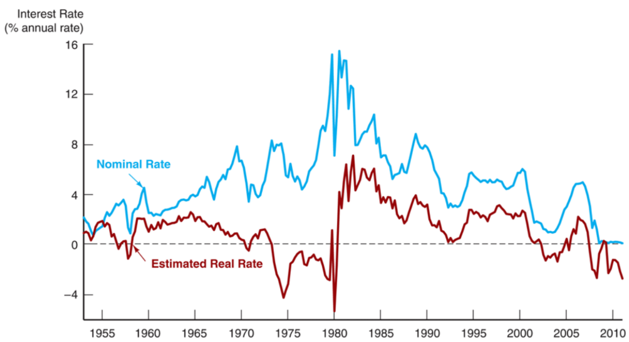
- if the two loan contracts with repayments
  - $P_t \times (1 + i)$
  - $P_t \times (1 + \pi) \times (1 + r)$

  are equal:

$$(1 + i) = (1 + \pi) \times (1 + r)$$
$$1 + i = 1 + r + \pi + r\pi$$
$$i = r + \pi + r\pi$$
$$i \approx r + \pi$$

if r and π are small the error by discarding $r\pi$ is very small, e.g. $r = 0.030$ and $\pi = 0.015$ results in $r\pi = 0.00045$, a less than one percent error.

Real & Nominal Interest Rates (3-Month T-Bill)



Estimating the real interest rate involves estimating expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.

If you expect the inflation rate to be 15% next year and a one-year bond has a yield to maturity of 7%, then the real interest rate on this bond is...?

Assuming the same coupon rate and maturity length, when the interest rate on a Treasury Inflation Protected Security is 3%, and the yield on a nonindexed Treasury bond is 8%, the expected rate of inflation is...?